

St. George's Independent School Collierville

Algebra II Summer Work Packet

Review and Study Guide

This study guide is designed to accompany the Algebra II Summer Work Packet. Its purpose is to offer a review of the ten specific concepts covered in the Summer Work Packet. The mastery of these concepts prior to beginning Algebra II is essential. For each concept, there are explanations and examples as well as extra problems that students may choose to do on their own if they are experiencing difficulty and would like to reassure themselves that they have indeed mastered the concepts. Students are not required to print out this Guide and may use it simply as an online reference as they complete their summer work. However, printing this packet with the intention of including it in their Algebra II binder as a reference guide would serve them well throughout the entire course.

Topics Covered in this Packet:

1. Integers
2. Fractions
3. Exponents
4. Radicals
5. Numbers vs. Variables
6. Order of Operations
7. Simplifying Algebraic Expressions
8. Solving Equations
9. Polynomials
10. Graphing

1. Integers

Integers are in the set of rational numbers which means that they can be expressed as a quotient (as long as the denominator is not zero) such that the decimal form of the quotient either terminates or repeats. The set of integers is the set of all whole numbers both positive and negative as well as zero.

Adding Integers:

To add integers, you must first look at their signs. If both the signs are the same, then you simply add the numbers and retain the same sign. If the signs are different, the sign of the larger number will be the sign of the answer and you will actually subtract the integers.

Example 1

Simplify.

$$(-2) + (-3)$$

$$= -5$$

signs are the same, so keep the sign and add the numbers

Example 2

Simplify.

$$-9 + 6$$

$$= -3$$

signs are different, so take the sign of the larger number and subtract

Subtracting Integers:

To subtract integers, you must understand that a subtraction sign is really just a negative sign. The subtraction sign will change the sign of the number that follows it. If you think of it this way, and change the sign of the following number, then you are really just adding integers. So, if you subtract a positive number, you are actually adding a negative number. If you subtract a negative number, you are actually adding a positive number. Let's look at some examples.

Example 1

Simplify.

$$(-9) - (-3)$$

$$= (-9) + 3$$

$$= -6$$

change the sign of the number that follows the subtraction sign and then add

Example 2

Simplify.

$$(-5) - 4$$

$$= (-5) + (-4)$$

$$= -9$$

here, the "4" was actually positive, so change the sign and then add

Example 3

Simplify.

$$4 - 7$$

$$= 4 + (-7)$$

$$= -3$$

here, the "7" was positive, so change the sign and follow the rules of addition

Multiplying/Dividing Integers:

To multiply or divide integers, you must apply the rules - you know the ones: A negative times a negative is a positive, a positive times a negative is a negative. So, you could really do multiplication/division of integers in two steps: First, multiply/divide the numbers; second, figure out what sign the answer will have

Example 1

Simplify.

$$\begin{aligned} &(-5) \times (-4) \\ &= 20 \end{aligned}$$

in the two step process, 4 times 5 is 20, and 2 negatives make a positive

Example 2

Simplify.

$$\begin{aligned} &(-10) \times 7 \\ &= -70 \end{aligned}$$

in the two step process, 10 times 7 is 70, and a neg. times a pos. is a neg.

Example 3

Simplify.

$$\begin{aligned} &(-10) \div (-2) \\ &= 5 \end{aligned}$$

Example 4

Simplify.

$$\begin{aligned} &20 \div (-2) \\ &= -10 \end{aligned}$$

Example 5

Simplify.

$$\begin{aligned} &-[-(-1)] \\ &= -1 \end{aligned}$$

a negative times a negative is a positive, times a negative is a negative

Try These:

Simplify.

1. $4 - 7 + 6$

2. $(-7) \times (-6)$

3. $(-6) \div 3$

4. $8 - 12 + 4$

5. $63 \div (-9)$

6. $14 - 7 - 3 - 16$

2. Fractions

Fractions are also called ratios. In an operational sense, they indicate that two numbers are to be divided. Fractions are also a good way of comparing two numbers. The top number of a fraction is the numerator and the bottom number of a fraction is the denominator. We are going to look at basic operations with fractions.

Simplified and Equivalent Fractions:

Everyone knows that one half is equal to three sixths, but not everyone understands why. If you multiply the top and the bottom of a fraction by the same number, you will get a new fraction that is

equivalent to your original fraction. In the example of $\frac{1}{2}$ and $\frac{3}{6}$, we multiplied both the top and the bottom by 3 and the resultant fraction is equivalent to $\frac{1}{2}$. We can also divide both the top and the bottom to reduce a fraction and get it down to a smaller, often more manageable number. Another process that is necessary for manipulating fractions is getting common denominators. This is necessary for adding, subtracting and comparing fractions. Because we can multiply the top and bottom by the same number and maintain the same fraction, we can force two fractions to have a common denominator. Let's do some:

Example 1

Get a common denominator for $\frac{1}{5}$ and $\frac{3}{4}$.

What is a number that both 5 and 4 can go into evenly?

What number are we going to use for our common denominator?

Well, as long as the resulting number is not too large, we can simply multiply the 2 denominators together. So, in this case, our common denominator is 20. But then what do we do with the numerators?

$\frac{?}{20}$ and $\frac{?}{20}$ to get 20 from 5 (first fraction), we multiplied by 4, so we must do the same in the numerator

$\frac{4}{20}$ and $\frac{15}{20}$ so we multiplied the first fraction by 4 (top and bottom) and the second fraction by 5 (top and bottom) and now they have a common denominator, and we can easily see that the first fraction is smaller

Example 2

Get a common denominator for $\frac{5}{12}$ and $\frac{3}{8}$

$\frac{?}{24}$ and $\frac{?}{24}$
 $\frac{10}{24}$ and $\frac{9}{24}$

Adding & Subtracting Fractions

To add or subtract fractions, you must always have a common denominator. Once the denominators are the same, you add or subtract only the numerator to get your final answer. The common denominator of choice is the *Lowest Common Denominator*. If you make an effort to keep the numbers in fractions as small as possible, it will make subsequent calculations much easier.

Example 1

Perform the indicated operation.

$$\begin{aligned}\frac{3}{4} - \frac{8}{16} \\ = \frac{3}{4} - \frac{2}{4} \\ = \frac{1}{4}\end{aligned}$$

(Note that in this case it is going to be much easier to have a common denominator of "4" rather than "32", "48" or "64".)

Example 2

Perform the indicated operation.

$$\begin{aligned}\frac{5}{6} + \frac{3}{8} \\ = \frac{20}{24} + \frac{9}{24} \\ = \frac{29}{24}\end{aligned}$$

Try These:

Perform the indicated operation.

1. $\frac{8}{9} - \frac{4}{5}$

2. $\frac{3}{4} + \frac{1}{6}$

3. $\frac{7}{10} - \frac{3}{8}$

4. $\left(\frac{3}{4} - \frac{1}{3}\right) + \frac{7}{12}$

5. $\frac{1}{2} + \frac{3}{5} + \frac{1}{10}$

6. $\frac{1}{5} + \frac{1}{6} - \frac{3}{15}$

Multiplying Fractions

Unlike adding and subtracting, you do not need a common denominator to multiply fractions. To multiply fractions you multiply the numerators and then the denominators. Then, it is good practice to always reduce your final answer. When multiplying a fraction by a whole number, you can rewrite the whole number in fraction form by putting a "1" in the denominator.

Example 3

Perform the indicated operation.

$$\begin{aligned}\frac{3}{4} \times \frac{2}{5} \\ = \frac{3 \times 2}{4 \times 5} \\ = \frac{6}{20} \\ = \frac{3}{10}\end{aligned}$$

Example 4

Perform the indicated operation.

$$\begin{aligned}4 \times \frac{3}{8} \\&= \frac{4}{1} \times \frac{3}{8} \\&= \frac{4 \times 3}{1 \times 8} \\&= \frac{12}{8} \\&= \frac{3}{2}\end{aligned}$$

Try These:

Perform the indicated operation.

1. $\frac{1}{2} \times \frac{4}{7}$

2. $\frac{7}{12} \times \frac{3}{4}$

3. $5 \left(\frac{1}{3} + \frac{2}{5} \right)$

4. $2 \left(\frac{3}{4} \right) \left(\frac{2}{7} \right)$

5. $\frac{4}{3} + \frac{6}{9} \times 4$

6. $\frac{3}{4} \times 8$

Dividing Fractions

You have heard this a thousand times: "Dividing by a fraction is the same as multiplying by its reciprocal." To get the reciprocal of a fraction, you switch the numerator and the denominator. Another way of thinking of it is: a fraction multiplied by its reciprocal will always give you "1". For example the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$, and $\frac{3}{5} \times \frac{5}{3}$ is $\frac{15}{15}$ or 1. As always, to maintain good form, you must reduce your final answer to its simplest form.

Example 1

Perform the indicated operation.

$$\begin{aligned}\frac{3}{4} \div \frac{2}{3} \\&= \frac{3}{4} \times \frac{3}{2} \\&= \frac{3 \times 3}{4 \times 2} \\&= \frac{9}{8}\end{aligned}$$

Example 2

Perform the indicated operation.

$$\begin{aligned}\frac{2}{3} \div 6 \\ &= \frac{2}{3} \div \frac{6}{1} \\ &= \frac{2}{3} \cdot \frac{1}{6} \\ &= \frac{2}{18} \\ &= \frac{1}{9}\end{aligned}$$

Try These: Perform the indicated operation.

1. $\frac{11}{18} \div \frac{1}{3}$

2. $\frac{2}{9} \div \frac{6}{7}$

3. $\frac{9}{10} \div \frac{5}{8}$

4. $\left(\frac{3}{7} \div \frac{6}{10}\right) \div \frac{4}{7}$

5. $\frac{4}{5} \times \frac{2}{3} \div \frac{8}{5}$

6. $6\left(\frac{11}{18} \div \frac{1}{3}\right)$

3. Exponents

The properties of exponents are as follows:

Power-of-a-Power:

$$(x^m)^n = x^{mn}$$

ex. $(7^2)^3 = 7^{2 \times 3} = 7^6 = 117649$

Power-of-a-Product:

$$(xy)^n = x^n y^n$$

ex. $(-3x)^2 = (-3)^2 (x)^2 = 9x^2$

Quotient-of-Powers:

$$\frac{x^m}{x^n} = x^{m-n}$$

ex. $\frac{3^8}{3^2} = 3^{8-2} = 3^6 = 729$

Positive Power of a Quotient:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

ex. $\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$

Negative Power of a Quotient:

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$

ex. $\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4 = \frac{3^4}{2^4} = \frac{81}{16}$

Zero Power:

$$a^0 = 1$$

$$\text{ex. } \left(\frac{x^3 y^7}{m^2 n^5} \right)^0 = 1$$

Example 1

Simplify each expression leaving no negative exponents.

$$\begin{aligned} & \frac{3x^5}{6x^{-2}} \\ &= \frac{(3x^5)(x^2)}{6} \\ &= \frac{3x^{10}}{6} \\ &= \frac{x^{10}}{2} \end{aligned}$$

Example 2

Simplify each expression leaving no negative exponents.

$$\begin{aligned} & 3x(2x^2y + 7y) \\ &= (3x)(2x^2y) + (3x)(7y) \\ &= 6x^3y + 21xy \end{aligned}$$

Example 3

Simplify each expression leaving no negative exponents.

$$\begin{aligned} & (4x^2)^{-3} \\ &= \frac{1}{(4x^2)^3} \\ &= \frac{1}{4^3 x^6} \\ &= \frac{1}{64x^6} \end{aligned}$$

Example 4

Simplify each expression leaving no negative exponents.

$$\begin{aligned} & \frac{4x^{-1}y^2}{5x^{-3}y^3} \times \frac{5y^4}{6y^{-3}} \\ &= \frac{4(x^3)(y^2)}{5(x)(y^3)} \times \frac{5(y^4)(y^3)}{6} \\ &= \frac{4x^2}{5y} \times \frac{5y^7}{6} \\ &= \frac{2x^2y^6}{3} \end{aligned}$$

Try These: Simplify each expression leaving no negative exponents.

1. $\left(\frac{s^3t}{st^4}\right)^2$

2. $\left(\left(\frac{3mn}{2}\right)^{-1}\right)^{-4}$

3. $\left(\frac{3c}{-2}\right)^{-1}\left(\frac{d}{4}\right)^{-2}$

4. $-\left(\frac{-t}{3v}\right)^{-4}$

5. $\left(\frac{6}{7}\right)^{-2}\left(\frac{4s}{6t}\right)^{-2}$

6. $\left(\frac{3a}{2b}\right)^{-4}$

4. Radicals

Some properties of radicals to know are:

Fractional Exponents: $x^{\frac{a}{b}} = (\sqrt[b]{x})^a$ ex. $8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = (2)^2 = 4$

Product Property: $\sqrt{ab} = \sqrt{a}\sqrt{b}$ ex. $\sqrt{32} = \sqrt{2 \times 16} = \sqrt{2} \times \sqrt{16} = \sqrt{2}(4) = 4\sqrt{2}$

Quotient Property: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ ex. $\sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}$

A few other things to remember about radicals:

► Only *like radicals* can be added and subtracted. *Like radicals* are radicals with the same *radicand* (number under the square root sign).

ex. $4\sqrt{2x} + 3\sqrt{2x} - \sqrt{2x} = 6\sqrt{2x}$

► A fraction should never include a radical in its denominator. It is good form to always *rationalize the denominator*.

ex. $\frac{4}{2\sqrt{3}} = \frac{4}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{2\sqrt{9}} = \frac{4\sqrt{3}}{(2)(3)} = \frac{2\sqrt{3}}{3}$

Example 1

Evaluate the expression.

$$\begin{aligned}(54)^{\frac{1}{2}} \\ &= \sqrt{54} \\ &= \sqrt{9} \sqrt{6} \\ &= 3\sqrt{6} \\ &= 3\sqrt{6}\end{aligned}$$

Example 2

Evaluate the expression.

$$\begin{aligned}\sqrt{x^3 y^6} \\ &= \sqrt{x(x \times x)(y \times y)(y \times y)(y \times y)} \\ &= x \times y \times y \times y \sqrt{x} \\ &= xy^3 \sqrt{x}\end{aligned}$$

Example 3

Evaluate the expression.

$$\begin{aligned}\left(x^{\frac{1}{4}}\right)^5 \left(x^{\frac{3}{4}}\right) \\ &= \left(x^{\frac{5}{4}}\right) \left(x^{\frac{3}{4}}\right) \\ &= x^{\frac{5}{4} + \frac{3}{4}} \\ &= x^{\frac{8}{4}} \\ &= x^2\end{aligned}$$

Example 4

Evaluate the expression leaving no radicals in the denominator.

$$\begin{aligned}\sqrt{\frac{x^2}{y}} \\ &= \frac{\sqrt{x^2}}{\sqrt{y}} \\ &= \frac{x}{\sqrt{y}} \\ &= \frac{x}{\sqrt{y}} \times \frac{\sqrt{y}}{\sqrt{y}} \\ &= \frac{x\sqrt{y}}{y}\end{aligned}$$

Try These: Evaluate the expression leaving no radicals in the denominator.

1. $\sqrt{18}$

2. $\sqrt{\frac{4x^5}{9}}$

3. $\sqrt{\frac{x^5}{25y^2}}$

4. $\sqrt{12} \times \sqrt{3}$

5. $\sqrt{\frac{72x^7}{4x^4}}$

6. $\sqrt{\frac{7x^4}{9x^3}}$

5. Numbers vs. Variables

Variables are letters used in algebraic expressions that stand for numbers. Although numbers and variables are very different, they are often used together. Therefore, it is important to understand how variables and numbers can be used together and what the rules are for combining them and separating them.

Adding/Subtracting Variables: Only two variables that look the same can be added or subtracted. For example, you cannot add $x^2 + x$ because these two variables (although they have the same base) are not the same (one is "squared" and the other is not). Before you can add or subtract, the variables themselves must look the same.

Example 1

Simplify.
 $x + x$
 $= 2x$

Example 2

Simplify.
 $5xy + 6x - 3xy$
 $= 5xy - 3xy + 6x$
 $= 2xy + 6x$

Example 3

Simplify.
 $5x - 7 + 2 + 4x$
 $= 5x + 4x - 7 + 2$
 $= 9x - 5$

Example 4

Simplify.
 $m + n - 7 + 8$
 $= 1 + m + n$

Multiplying Variables: A number and a variable can be multiplied together to form one term. Also a variable and another variable can be multiplied to form one term.

Example 1

Simplify.

$$\begin{aligned} & y \times 3 \times 6 \\ & = y \times 18 \\ & = 18y \end{aligned}$$

Example 2

Simplify.

$$\begin{aligned} & 4 \times m \times 7 \\ & = 4 \times 7 \times m \\ & = 28m \end{aligned}$$

Example 3

Simplify.

$$\begin{aligned} & m \times n \times m \times b \\ & = m^2nb \end{aligned}$$

Try These:

1. $4 \times 3 \times 2 \times y \times z$

2. $\frac{8xy}{2x}$

3. $4n + 5m + n - m - 2m$

4. $m + m$

5. $m \times m$

6. $2(m \times m)$

6. Order of Operations

The rules for Order of Operations are as follows:

FIRST: Perform operations inside grouping symbols. Grouping symbols include parentheses (), brackets [], braces { }, radical symbols $\sqrt{\quad}$, absolute value symbols $|\quad|$ and fraction bars. If an expression contains more than one set of grouping symbols, simplify the expression inside the innermost set first. Follow the order of operations within that set of grouping symbols and then work outward.

SECOND: Simplify exponents.

THIRD: Perform multiplication and division from left to right. (Remember that a fraction bar also indicates division.)

FOURTH: Perform addition and subtraction from left to right.

Hint: You can use the well-known phrase "Please Excuse My Dear Aunt Sally" to help you remember the Order of Operations. (Remember, however, that multiplication and division must be done in the order that they appear if they do not appear in parentheses. This is also true for addition and subtraction.)

P lease	P arentheses
E xcuse	E xponents
M y	M ultiplication
D ear	D ivision
A unt	A ddition
S ally	S ubtraction

Example 1

Simplify.

$$\begin{aligned}
 & -4^2 + 24 \div 3 \times 2 \\
 & = -16 + 24 \div 3 \times 2 \\
 & = -16 + 8 \times 2 \\
 & = -16 + 16 \\
 & = 0
 \end{aligned}$$

(Note that there are no grouping symbols. Therefore the exponent only applies to the "4" and not the "-".)

Example 2

Simplify.

$$\begin{aligned}
 & 4[25 - (5 - 2)^2] \\
 & = 4[25 - (3)^2] \\
 & = 4[25 - 9] \\
 & = 4 \times 6 \\
 & = 64
 \end{aligned}$$

Example 3

Simplify.

$$\begin{aligned}
 & \frac{-22 - 2^2}{5 - 3} \\
 & = \frac{(-22 - 2^2)}{(5 - 3)} \\
 & = \frac{-22 - 4}{2} \\
 & = \frac{-26}{2} \\
 & = -13
 \end{aligned}$$

(Note that the fraction bar acts as a grouping symbol. You must simplify the numerator and denominator before dividing.)

Try These: Simplify.

1. $8 \div \frac{1}{2} \times 3$

2. $-20 \div [-2(4+1)]$

3. $\frac{5+2(-8)}{(-2)^3-3}$

4. $3\sqrt{50-1}$

5. $|4-7|^2 \div (-3)$

6. $\frac{2(-4)+22}{4^2-9}$

7. Simplifying Algebraic Expressions

To simplify algebraic expressions you need to apply the rules for the Order of Operations and collect *like terms*. *Like terms* are terms that contain the same variables raised to the same powers. Constants (numbers with no variable) are also *like terms* and can be simplified according to the order of operations. Typically, simplified expressions are written with the variables in descending order according to their exponents.

Distributive Property:

$$a(b+c) = ab+ac$$

ex. $3(x-4) = 3x-12$

► When simplifying algebraic expressions it is important to remember the Algebraic Properties of Equality. The Distributive Property is important and is also the property with which many students have difficulties. Remember that when the coefficient in front of parentheses is preceded by a negative, that negative must also be distributed through the parentheses.

For example if we apply the distributive property to $x+4(x-3)$, the 4 is going to be distributed through the parentheses to get $x+4x-12$ or $5x-12$. However if we apply the distributive property to $x-4(x-3)$, the -4 must be distributed to get $x-4x-12$ or $-3x-12$.

Example 1

Simplify.

$$\begin{aligned} &4x(1-x) - (3x^2 - 2x + 7) \\ &= 4x - 4x^2 - 3x^2 + 2x - 7 \\ &= 6x - 2x^2 - 7 \\ &= -2x^2 + 6x - 7 \end{aligned}$$

Example 2

Simplify.

$$\begin{aligned} &(x-2y) - 2(5x-y-7) \\ &= x - 2y - 10x + 2y + 14 \\ &= -9x + 14 \end{aligned}$$

Example 3

Simplify.

$$\begin{aligned} & \left[\frac{4(x^4 - x^2) + 12x^4 + 4x^2}{4x^2} \right]^{1/2} \\ &= \left(\frac{4x^4 - 4x^2 + 12x^4 + 4x^2}{4x^2} \right)^{1/2} \\ &= \left(\frac{16x^4}{4x^2} \right)^{1/2} \\ &= (4x^2)^{1/2} \\ &= \sqrt{4x^2} \\ &= 2x \end{aligned}$$

Try These: Simplify.

1. $5(x+3) - 7x$

2. $12x + 8x + y - 7x$

3. $\frac{\frac{2}{5}x^3 + \frac{1}{5}x^3 - \frac{1}{2}(3x^2 - 6x^2)}{x}$

4. $24x + y^2 + 3x + 2y^2$

5. $4[3(x+9) + 2] - 7x$

6. $\frac{1}{2}[(10-x) - (-6+3x)]$

8. Solving Equations

To solve equations, we must isolate the variable in the equation to find the value of the variable that makes the equation true. We can think of an equation as a balanced scale. We must perform the same operation to both sides of the equation to keep it equal. When solving equations remember: "What you do to one side, you must do to the other." When solving linear equations, $0 = 0$ is an acceptable answer and is translated to mean that "x" can be any real number.

Example 1

Solve for x.

$$2(2x - 1) = 3x + 4$$

$$4x - 2 = 3x + 4$$

$$4x - 3x = 4 + 2$$

$$x = 6$$

Example 2

Solve for x .

$$\frac{2x+3}{5} = \frac{x-1}{3} \quad \text{This is a proportion and can be solved by cross-multiplying.}$$

$$3(2x+3) = 5(x-1)$$

$$6x+9 = 5x-5$$

$$6x-5x = -5-9$$

$$x = -14$$

Example 3

Solve for x .

$$4x^2 = 36$$

$$x^2 = 9$$

$$x = \pm 3$$

Remember that there are two answers here. Since "x" is squared, its value can be both positive and negative.

Try These: Solve for x .

1. $6-7(x+1) = -3(2-x)$

2. $-5-7-3x = -x-2(x+6)$

3. $\frac{2}{3}x + \frac{1}{2} = \frac{3}{5}x - \frac{5}{6}$

4. $\frac{3x-5}{4} = \frac{x-5}{3}$

5. $4(3x+1) - 7x = 6+5x-2$

6. $4x+2[4-2(x+2)] = 2x-4$

9. Polynomial Expressions & Equations

A polynomial is defined as a monomial or a sum or difference of monomials.

F.O.I.L. (**First, Outer, Inner, Last**) FOIL is the method by which two binomials can be multiplied using the distributive property.

Example 1

Multiply the binomials using FOIL.

$$(x+3)(x+2) \quad \text{Take the first term of the first binomial and distribute it through the second binomial.}$$

Then, take the second term in the first binomial and distribute it through the second binomial.

$$= (x \times x) + (x \times 2) + (3 \times x) + (3 \times 2)$$

$$= x^2 + 2x + 3x + 6$$

$$= x^2 + 5x + 6$$

Example 2

Multiply the binomials using FOIL.

$$(x-8)(7x+4)$$

$$= (x \times 7x) + (x \times 4) + (-8 \times 7x) + (-8 \times 4)$$

$$= 7x^2 + 4x - 56x - 32$$

$$= 7x^2 - 52x - 32$$

10. Linear Graphs

Slope of a Line: The rate of change that compares the amount of change in a dependent variable (y) to the amount of change in an independent variable (x).

$$\text{slope} = \frac{\text{change in dependent variable}}{\text{change in independent variable}} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

A Few Things to Remember:

- ▶ Two lines that are parallel have the same slope.
- ▶ Two lines that are perpendicular have slopes that are negative reciprocals.
- ▶ The slope of a horizontal line is 0.
- ▶ The slope of a vertical line is undefined.
- ▶ Slope-Intercept form is $y = mx + b$.
- ▶ Standard form is $Ax + By = C$.

Example 1 Given the two points M(4, 2) and N(-2, -1) on a line, find the slope of the line \overline{MN} .

$$\begin{aligned}\text{slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - (-1)}{4 - (-2)} \\ &= \frac{2 + 1}{4 + 2} \\ &= \frac{3}{6} \\ &= \frac{1}{2}\end{aligned}$$

Intercepts: The x-intercept is the point at which the graph crosses the x-axis. (In other words, it's the point at which $y = 0$.) The y-intercept is the point at which the graph crosses the y-axis. (In other words, it's the point at which $x = 0$.) Using intercepts is one of the easiest ways to graph a line on the coordinate plane.

ex. Let's say you want to graph the line defined by:

$$3x - 2y = 12$$

To find the x-intercept, set $y = 0$ and solve for x :

$$3x - 2(0) = 12$$

$$3x = 12$$

$$x = 4$$

Now, on the coordinate plane, go to 4 on the x-axis to graph the point at which this graph crosses the x-axis.

To find the y-intercept, set $x = 0$ and solve for y :

$$3(0) - 2y = 12$$

$$2y = 12$$

$$y = 6$$

Now, on the coordinate plane, go to 6 on the y-axis to graph the point at which this graph crosses the y-axis.

Connect the x- and y-intercepts and you have the graph of the line $3x - 2y = 12$

Example 2 Find the intercepts for the line $\frac{1}{2}x = 4y + 4$.

x-int.

$$\frac{1}{2}x = 4(0) + 4$$

$$\frac{1}{2}x = 4$$

$$x = 8$$

x-int. is (8, 0)

y-int.

$$\frac{1}{2}(0) = 4y + 4$$

$$0 = 4y + 4$$

$$4y = -4$$

$$y = -1$$

y-int. is (0, -1)

Slope-Intercept Form

The Slope-Intercept form of the equation of a line is:

$$y = mx + b$$

Writing the equation of a line in this form gives you a lot of valuable information immediately because in this equation m is the slope of the line and b is the y-intercept.

Try These:

Find the slope of the line containing the two given points. Also, state the slope of a line that is perpendicular.

1. M(-1, 4), N(-1, -3)

2. M(0, 2), N(4, -1)

3. M(-6, 3), N(1, -4)

Find the slope as well as the x- and y-intercepts of the given lines.

4. $y = 4 - 2x$

5. $-2x - y = 4$

6. $x - 3y = -1$