

ADVANCED TOPICS IN CALCULUS AND DISCRETE MATH - SUMMER PACKET

Complete all steps of each problem on separate paper. Be neat and attempt all problems. This will be graded as a Test grade and is due the first full day of school. Part of your grade will be based on effort and the remainder of the grade on correctness. The internet is a great resource; use it. Feel free to contact me at asthompson@sgis.org if you need help finding a useful website/resource or have questions.

1. Are the following statements true? If not, change them to make them true.

a. $\frac{2k}{2k+4} = \frac{k}{x+4}$

b. $\frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$

c. $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$

d. $3\left(\frac{a}{b}\right) = \frac{3a}{3b}$

e. $3\left(\frac{a}{b}\right) = \frac{3a}{b}$

f. $3\left(\frac{a+b}{c}\right) = \frac{3a+b}{c}$

2. Simplify.

a. $\frac{\frac{x}{2}}{\frac{x}{4}}$

b. $h \div \frac{x+h}{h}$

c. $\frac{\sqrt{x-2} + \frac{5}{\sqrt{x-2}}}{x-2}$

d. $\frac{x^3}{x^{-5}}$

e. $\frac{2x^3}{y^{-5}} \cdot \frac{y^2}{3x^7}$

f. $\frac{x^2-4x-5}{x^2+2x+1}$

g. $\frac{x-4}{4-x}$

h. $(x-1)^3$

i. $x^{\frac{1}{3}}x^{\frac{3}{5}}$

3. Solve the equation for all real values of x.

a. $4x^2-21x-18=0$

b. $2x^2-3x+3=0$

c. $x^4-9x^2+8=0$

d. $\frac{2}{x+1} = \frac{x-2}{2}$

e. $x^2-9x+9=0$

f. $\frac{1}{x} + x = 4$

g. $\frac{5}{e^x+1} = 1$

h. $\sqrt{x-1} - \frac{5}{\sqrt{x-1}} = 0$

4. Write as a single fraction with the denominator in factored form.

a. $\frac{7x^2+5x}{x^2+1} - \frac{5x}{x^2-6}$

b. $20\left(\frac{2}{x+1} - \frac{3}{x}\right)$

5. Graph the equation $y = x^3 - x$ and answer the following questions.

a. Is the point (3, 2) on the graph?

b. Is the point (2, 6) on the graph?

c. Is the function odd, even or neither?

d. Find the x and y - intercept(s).

6. Factor completely.

a. $3x^3+192$

b. $9x^2-3x-2$

7. Find the equation of the line that passes through the point (2, 4) and is parallel to the line $2x + 3y - 8 = 0$.

8. Find the equation of the line that is perpendicular to the line $2x + 3y - 8 = 0$ at the point $(1, 2)$.
9. The line with slope 5 that passes through the point $(-1, 3)$ intersects the x-axis at a point. What are the coordinates of this point?
10. What are the coordinates of the point at which the line passing through the points $(1, -3)$ and $(-2, 4)$ intersects the y-axis?
11. A 20 foot ladder rests against a building 15 feet from the floor. How far does the ladder extend from the base of the wall? What angle does the ladder make with the ground?

12. Given $f(x) = x - 3$ and $g(x) = \sqrt{x}$, complete the following.

a. $f(g(x)) =$

b. $g(f(x)) =$

c. $f(f(x)) =$

13. Given $f(x) = \frac{1}{x-5}$ and $g(x) = x^2 - 5$, complete the following.

a. $f(g(7)) =$

b. $g(f(v)) =$

c. $g(g(x)) =$

14. Find all intercepts and asymptotes.

a. $y^2 = x^2 - 4x$

b. $y = \frac{x^2 + 3x}{(3x+1)^2}$

c. $y = \frac{x^2 - 4}{x^2 - x - 12}$

d. $y = \frac{3x-1}{2x^2+x-6}$

15. A seven foot ladder, leaning against a wall, touches the wall x feet above the ground. Write an expression in terms of x for the distance from the foot of the ladder to the base of the wall.

16. A piece of wire 5 inches long is to be cut into two pieces. One piece is x inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of x .

17. Solve the following for the principal values of the indicated variable.

a. $3\cos x - 1 = 2$

b. $2\sin(2x) - \sqrt{3} = 0$

c. $\tan^2 x - 1 = 0$

18. Evaluate.

a. $\cos 0$

b. $\sin 0$

c. $\tan \frac{\pi}{2}$

d. $\cos \frac{\pi}{4}$

e. $\sin \frac{\pi}{2}$

f. $\sin \pi$

g. $\sin^{-1} \frac{\sqrt{3}}{2}$

h. $\tan^{-1} 1$

i. $\cos^{-1} \frac{1}{2}$

j. $\sec^{-1} \sqrt{2}$

k. $\cos^{-1}(-1)$

l. $\sec \frac{\pi}{2}$

m. $\tan\left(-\frac{\pi}{6}\right)$

n. $\sin \frac{5\pi}{3}$

o. $\csc\left(-\frac{9\pi}{4}\right)$

p. $\cos(-3\pi)$

q. $\tan^{-1}(-1)$

YOU MAY USE A CALCULATOR FOR #19 – 24 FOR ARITHMETIC ONLY.

19. A police car receives a radio call to catch a vehicle which is speeding down the highway at 80 mph. The police car, which is 12 miles away, drives after it at 108 mph. How long will it take for the police car to catch up?
20. The base of a triangle is 6 cm more than the height. If the area of the triangle is 140 square cm, what is the length of the base?
21. Two trains, the Express and the Commuter, leave the same station at the same time. The Express, which heads north, travels 10 km per hour faster than the Commuter, which goes east. If the trains are 100 km apart after 2 hours, find the speed of each train.
22. On January 1st 2003, the value of a stock was \$135 per share. By December 1st 2003, the value of the stock had fallen to \$38 per share. What is the average rate of change in the value of the stock in dollars per month?
23. During a recent trip to the store, a car's velocity went from 0 to 60 mph in 20 seconds. What is the average acceleration of the car in miles per hour per hour?
24. Graph each.

a. $y = \begin{cases} 3-x & x \leq 1 \\ 2x & x > 1 \end{cases}$

b. $y = \begin{cases} 4-x^2 & x < 1 \\ \frac{3}{2}x + \frac{3}{2} & 1 \leq x \leq 3 \\ x+2 & x > 3 \end{cases}$