AP CALCULUS BC - SUMMER PACKET

Complete all steps of each problem on separate paper. Be neat and attempt all problems. This will be graded as a Test grade and is due the first full day of school. Part of your grade will be based on effort and the remainder of the grade on correctness. The internet is a great resource; use it. Feel free to contact me at asthompson@sgis.org if you need help finding a useful website/resource or have questions.

1. Are the following statements true? If not, change them to make them true.

   a. $\frac{2k}{2k+4}=\frac{k}{x+4}$ b. $\frac{1}{p+q}=\frac{1}{p}+\frac{1}{q}$ c. $\frac{x+y}{2}=\frac{x}{2}+\frac{y}{2}$

   d. $3\left(\frac{a}{b}\right)=\frac{3a}{3b}$ e. $3\left(\frac{a}{b}\right)=\frac{3a}{b}$ f. $3\left(\frac{a+b}{c}\right)=\frac{3a+b}{c}$

2. Simplify.

   a. $\frac{\frac{x}{2}}{\frac{x}{4}}$ b. $h\div\frac{x+h}{h}$ c. $\frac{\sqrt{x-2}+\frac{5}{\sqrt{x-2}}}{x-2}$ d. $\frac{x^3}{x^{-5}}$ e. $\frac{2x^3}{y^{-5}}\cdot\frac{y^2}{3x^7}$

   f. $\frac{x^2-4x-5}{x^2+2x+1}$ g. $\frac{x-4}{4-x}$ h. $(x-1)^3$ i. $x^{\frac{1}{3}}x^{\frac{3}{5}}$

   j. $\ln 5+\ln\left(x^2-1\right)-\ln(x-1)$ k. $\frac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$ l. $3^{2\log_3 5}$

3. Solve for y′.

   a. $xy' + y = 1 + y'$ b. $3y^2y' + 2yy' = 5y' + 2x$ c. $3x^2yy' + 2xy^2 = 2yy'$

4. Solve the equation for all real values of x.

   a. $4x^2-21x-18=0$ b. $2x^2-3x+3=0$ c. $x^4-9x^2+8=0$ d. $\frac{2}{x+1}=\frac{x-2}{2}$

   e. $x^2-9x+9=0$ f. $\frac{1}{x}+x=4$ g. $\frac{5}{e^x+1}=1$ h. $\sqrt{x-1}-\frac{5}{\sqrt{x-1}}=0$

   i. $5^{(x+1)}=\frac{1}{25}$ j. $\log x=3$ k. $\log_3 x^2=2\log_3 4-4\log_3 5$

   l. $2x+1=\frac{5}{x+2}$ m. $e^{\ln x-3\ln x}=\frac{5}{2}$ n. $\ln(2x)-\ln(x-3)=0$

5. Write as a single fraction with the denominator in factored form.

   a. $\frac{7x^2+5x}{x^2+1}-\frac{5x}{x^2-6}$ b. $20\left(\frac{2}{x+1}-\frac{3}{x}\right)$

6. Graph the equation $y = x^3 - x$ and answer the following questions.
   a. Is the point (3, 2) on the graph?
   b. Is the point (2, 6) on the graph?
   c. Is the function odd, even or neither?
   d. Find the x and y – intercept(s).

7. Factor completely.

   a. $3x^3+192$ b. $9x^2-3x-2$ c. $2\sqrt{x}-6x^{\frac{3}{2}}$

   d. sinx + tanx e. $e^{-x}-xe^{-x}+2x^2e^{-x}$

8. Find the equation of the line that passes through the point (2, 4) and is parallel to the line $2x + 3y – 8 = 0$.

9. Find the equation of the line that is perpendicular to the line $2x + 3y – 8 = 0$ at the point (1, 2).

10. The line with slope 5 that passes through the point (-1, 3) intersects the x-axis at a point. What are the coordinates of this point?

11. What are the coordinates of the point at which the line passing through the points (1, -3) and (-2, 4) intersects the y-axis?

12. A 20 foot ladder rests against a building 15 feet from the floor. How far does the ladder extend from the base of the wall? What angle does the ladder make with the ground?

13. Find f(1) – f(5) given $f(x) = |x – 3| - 5$.

14. Find f(x + 2) – f(2) given f(x) = x^2-3x+4.

15. Use interval notation to identify the domain for each of the following functions.

   a. $h(x)=\dfrac{1}{4x^2-21x-18}$ b. $k(x)=\sqrt{x^2-5x-14}$ c. $\dfrac{\sqrt[3]{x-6}}{\sqrt{x^2-x-30}}$ d. $d(x)=\ln(2x-12)$

16. Find f(x + h) for f(x) = x^2-2x-3.

17. Find $\dfrac{f(x+h)-f(x)}{h}$ if f(x) = $8x^2+1$.

18. Find $\dfrac{f(x+h)-f(x)}{h}$ if $f(x)=\dfrac{1}{x}$.

19. Determine whether the following functions are even, odd, or neither.

   a. $f(x) = x^5 – x$ b. $f(x) = x^6 - 8x^2 + 4$ c. $f(x) = 3x^3 – 1$

20. Let F(x) be an even function and G(x) be an odd function. Use the definition of even and odd functions to determine whether the composite functions below are even, odd or neither.

a. F(x)G(x) b. F(x)/G(x) c. F(G(x)) d. G(F(x)) e. F(x) - G(x)

21. Graph the function.

a. f(x) = $\begin{cases} 1 & x \le 0 \\ -1 & x > 0 \end{cases}$

b. f(x) = $\begin{cases} 2x & (-\infty, -1) \\ 2x^2 & [-1, 2) \\ -x+3 & [2, \infty) \end{cases}$

c. f(x) = $\sqrt{16 - x^2}$

22. Given f(x) = x – 3 and g(x) = $\sqrt{x}$, complete the following.

a. f(g(x)) = b. g(f(x)) = c. f(f(x)) =

23. Given $f(x) = \dfrac{1}{x-5}$ and g(x) = $x^2 - 5$, complete the following.

a. f(g(7)) = b. g(f(v)) = c. g(g(x)) =

24. Find all intercepts and asymptotes.

a. $y^2 = x^2 - 4x$ b. $y = \dfrac{x^2 + 3x}{(3x+1)^2}$ c. $y = \dfrac{x^2 - 4}{x^2 - x - 12}$

d. $y = \dfrac{3x-1}{2x^2 + x - 6}$

25. Simplify using only positive exponents. Do not rationalize the denominators.

a. $\dfrac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$ b. $\left(\dfrac{1}{x^{-2}} + \dfrac{4}{x^{-1}y^{-1}} + \dfrac{1}{y^{-2}}\right)^{-\frac{1}{2}}$ c. $\left(\dfrac{x^{-2}}{y^{-1}} - x\right)^{-3}$

26. Simplify.

a. $x(1-2x)^{-\frac{3}{2}} + (1-2x)^{-\frac{1}{2}}$ b. $(3x-2)^{\frac{1}{2}} + x(3x-2)^{-\frac{1}{2}}$ c. $\dfrac{\dfrac{2}{x} - 3}{1 - \dfrac{1}{x-1}}$

d. $2\ln(x-3) + \ln(x+2) - 6\ln x$

27. Find the surface area of a box of height *h* whose base dimensions are *p* and *q* and satisfies the following conditons.

a. The box is closed.
b. The box has an open top.
c. The box has an open top and a square base with side length *p*.

28. A seven foot ladder, leaning against a wall, touches the wall x feet above the ground. Write an expression in terms of x for the distance from the foot of the ladder to the base of the wall.

29. A piece of wire 5 inches long is to be cut into two pieces. One piece is x inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the spare and the circle as a function of x.

30. Solve the following for the principal values of the indicated variable.

a. $3\cos x - 1 = 2$ b. $2\sin(2x) - \sqrt{3} = 0$ c. $\tan^2 x - 1 = 0$

31. Evaluate.

a. $\cos 0$ b. $\sin 0$ c. $\tan \frac{\pi}{2}$ d. $\cos \frac{\pi}{4}$ e. $\sin \frac{\pi}{2}$

f. $\sin \pi$ g. $\sin^{-1} \frac{\sqrt{3}}{2}$ h. $\tan^{-1} 1$ i. $\cos^{-1} \frac{1}{2}$ j. $\sec^{-1} \sqrt{2}$

k. $\cos^{-1}(-1)$ l. $\sec \frac{\pi}{2}$ m. $\tan\left(-\frac{\pi}{6}\right)$ n. $\sin \frac{5\pi}{3}$ o. $\csc\left(-\frac{9\pi}{4}\right)$

p. $\cos(-3\pi)$ q. $\tan^{-1}(-1)$

32. Express y in terms of x.

a. $\ln y = x + 2$ b. $\ln y = 2\ln x + \ln 10$ \

c. $\ln y = 4\ln x + 3$ d. $x = \ln \frac{e^{x^2}}{4y}$

33. Which of the following expressions are identical?

a. $\cos^2 x$ b. $(\cos x)^2$ c. $\cos x^2$

34. Which of the following expressions are identical?

a. $(\sin x)^{-1}$ b. arcsinx c. $\sin x^{-1}$ d. $\frac{1}{\sin x}$

35. Solve for x.

a. $\ln e^3 = x$ b. $\ln e^x = 4$ c. $\ln x + \ln x = 0$ d. $e^{\ln 5} = x$

e. $\ln 1 - \ln e = x$ f. $\ln 6 + \ln x - \ln 2 = 3$ g. $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$

YOU MAY USE A CALCULATOR FOR #36 – 44 FOR ARITHMETIC ONLY.

36. A police care receives a radio call to catch a vehicle which is speeding down the highway at 80 mph. The police car, which is 12 miles away, drives after it at 108 mph. How long will it take for the police car to catch up?

37. The base of a triangle is 6 cm more than the height. If the area of the triangle is 140 spare cm, what is the length of the base?

38. Two trains, the Express and the Commuter, leave the same station at the same time. The Express, which heads north, travels 10 km per hour faster than the Commuter, which goes east. If the trains are 100 km apart after 2 hours, find the speed of each train.

39. The depth, d, of a buoyant object t seconds after plunging into water can be found using the equation $d = -6t^2 + rt$, where r is the velocity at which the object strikes the water. If the object strikes the water at a velocity of 240 feet per second, find the maximum depth the rocket will reach and at what time. When will the rocket surface again?

40. Find the average rate of change for the following functions on the indicated intervals.
a. $f(x) = x^3 - 2x; [0, 4]$ b. $f(x) = 3\sqrt{x}; [4, 25]$

41. A car travels 420 miles over a period of 210 minutes. Find the average velocity of the car in miles per hour over this time period.

42. On January 1st 2003, the value of a stock was \$135 per share. By December 1st 2003, the value of the stock had fallen to \$38 per share. What is the average rate of change in the value of the stock in dollars per month?

43. In 1984, the Fizzy Cola company sold 23 million gallons of soda. By 2003, the company was selling 127 million gallons of soda. What is the average rate of change in the number of gallons of soda sold per year?

44. During a recent trip to the store, a car's velocity went from 0 to 60 mph in 20 seconds. What is the average acceleration of the car in miles per hour per hour?

45. Graph each.

a. $y = \begin{cases} 3-x & x \le 1 \\ 2x & x > 1 \end{cases}$ b. $y = \begin{cases} 4-x^2 & x < 1 \\ \frac{3}{2}x + \frac{3}{2} & 1 \le x \le 3 \\ x+2 & x > 3 \end{cases}$ c. $y = \frac{|x+1|}{x+1}$ d. $y = |x-2| + 3$

46. Evaluate the limit. Verify on your calculator by looking at the graph and using the trace key.

a. $\lim_{x \to 3} (x^2 + 2)$ b. $\lim_{x \to -3} \frac{(x+3)(x-4)}{(x+3)(x+1)}$ c. $\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25}$ d. $\lim_{x \to -2} \frac{x-4}{x^2 - 2x - 8}$

e. $\lim_{x \to -2} \frac{x^2 + 2x - 3}{x^2 + 7x + 12}$ f. $\lim_{x \to -2} \frac{x^3 + 8}{x + 2}$ g. $\lim_{x \to 5} \frac{x-5}{|x-5|}$ h. $\lim_{x \to 8} \frac{1}{x-8}$

47. For each of the following, determine: i. $\lim_{x \to 1^-} f(x)$ ii. $\lim_{x \to 1+} f(x)$ iii. $\lim_{x \to 1} f(x)$.

a. $y = \begin{cases} 3-x & x \le 1 \\ 2x & x > 1 \end{cases}$ b. $y = \begin{cases} 3x-1 & x \le 1 \\ 2-x & x > 1 \end{cases}$ c. $y = \begin{cases} -x^2 & x < 1 \\ 2 & x = 1 \\ x-2 & x > 1 \end{cases}$

48. Evaluate the following limits.

a. $\lim_{x\to 2}\dfrac{x^2-7x+10}{x^2-4}$

b. $\lim_{x\to -1}\dfrac{x^2+x-2}{x^2-1}$

c. $\lim_{x\to 5}\dfrac{x^2+2x-35}{x^2-10x+25}$

d. $\lim_{x\to 25}\dfrac{5-\sqrt{x}}{25-x}$

e. $\lim_{x\to 9}\dfrac{9-x}{\sqrt{x}-3}$

f. $\lim_{x\to 0}\dfrac{(x+3)^3-27}{x}$

g. $\lim_{x\to 0}\dfrac{x^2}{\sqrt{x^2+12}-\sqrt{12}}$

h. $\lim_{x\to 0}\dfrac{3}{x}\left(\dfrac{1}{5+x}-\dfrac{1}{5-x}\right)$

i. $\lim_{x\to 4}\dfrac{(x-4)^3}{|4-x|}$

49. For each of the following functions, find the value of c which makes the function continuous.

a. $y=\begin{cases}2x+c & x\le 1\\ x^2+3 & x>1\end{cases}$

b. $y=\begin{cases}cx+5 & x\le 2\\ 7x-c & x>2\end{cases}$

c. $y=\begin{cases}|2x+c| & x<-3\\ 1-x & x\ge -3\end{cases}$

d. $y=\begin{cases}\sin(x+c) & x<1\\ x^2-1 & x\ge 1\end{cases}$

e. $y=\begin{cases}ce^{3x} & x\ge -1\\ x^3+x+1 & x<-1\end{cases}$

f. $y=\begin{cases}(x+c)^2 & x<3\\ 5x+c & x\ge 3\end{cases}$

g. $y=\begin{cases}(x-2c)^2 & x<4\\ 2x+2c & x\ge 4\end{cases}$

h. $y=\begin{cases}e^{2x+c} & x\ge 0\\ x+2 & x<0\end{cases}$